

# Symmetries and solvable models for evaporating 2D black holes\*

J. Cruz, J. Navarro-Salas<sup>a</sup>, M. Navarro<sup>b</sup> and C. F. Talavera<sup>c</sup>

<sup>a</sup>Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC, Facultad de Física, Universidad de Valencia, Burjassot, 46100, Valencia, Spain.

<sup>b</sup>Instituto de Matemáticas y Física Fundamental, CSIC, Serrano 113-123, 28006 Madrid, Spain.

<sup>c</sup>Departamento de Matemática Aplicada, E.T.S.I.I., Universidad Politécnica de Valencia, Camino de Vera, 46100, Valencia, Spain.

We study the evaporation process of a 2D black hole in thermal equilibrium when the ingoing radiation is switched off suddenly. We also introduce global symmetries of generic 2D dilaton gravity models which generalize the extra symmetry of the CGHS model.

## 1. INTRODUCTION

In recent years it has been a lot of activity in the study of two-dimensional dilaton gravity models. The string-inspired model of Callan, Giddings, Harvey and Strominger [1] is the simplest model that describes the formation of black holes by gravitational collapse. The existence of an extra symmetry for the CGHS model plays a fundamental role at the classical and quantum level. The one-loop quantum corrected model of Russo, Susskind and Thorlacius [2] was constructed to preserve the free field associated with the extra symmetry and the classical ground state. The model of Bose, Parker and Peleg [3] also preserves the extra symmetry but describes an evaporating black hole with a non-flat remnant geometry.

The content of this work has two folds. In Sections 2 and 3 we shall study the one-parameter family of models interpolating between the BPP and RST models [4] and the exponential model [5][6] respectively. However, instead of studying the evaporation of a black hole formed by gravitational collapse we shall analyze the semiclassical evaporation of a black hole initially in thermal equilibrium when the incoming thermal flux is switched off suddenly. In Section 4 we intro-

duce global extra symmetries for generic 2D dilaton gravity models, aiming to generalize the extra symmetry of the one-loop corrected theories of the CGHS model.

## 2. THERMAL BATH REMOVAL IN THE BPP-RST MODELS

Let us consider the one-parameter family of models interpolating between the BPP and RST models [4],

$$S = S_0 + S_P + \frac{N}{24\pi} \int d^2x \sqrt{-g} [(1-2\phi)(\nabla\phi)^2 + (a-1)\phi R], \quad (1)$$

where  $S_0$  is the classical CGHS action

$$S_0 = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ e^{-2\phi} (R + 4(\nabla\phi)^2 + 4\lambda^2) - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right], \quad (2)$$

$S_P$  is the Polyakov term and  $a$  is an arbitrary real parameter. In the conformal ( $ds^2 = -e^{2\rho} dx^+ dx^-$ ) and Kruskal gauge ( $\rho = \phi$ ) it is very easy to see that the above models admit the following solution

$$e^{-2\phi} + N \frac{a}{12} \phi = -\lambda^2 x^+ x^- + \frac{M}{\lambda}, \quad (3)$$

\*Work partially supported by the *Comisión Interministerial de Ciencia y Tecnología* and *DGICYT*.

which represents a black hole in thermal equilibrium at temperature  $T = \frac{\lambda}{2\pi}$ . We can now remove the thermal bath by introducing a discontinuity in the boundary condition [7]

$$t_{x^+} = \frac{1}{4(x^+)^2} \theta(x^+ - x_0^+), \quad (4)$$

$$t_{x^-} = 0. \quad (5)$$

In this way we get the dynamical solution (for  $x^+ > x_0^+$ )

$$e^{-2\phi} + N \frac{a}{12} \phi = -\lambda^2 x^+ (x^- + \Delta) - \frac{N}{48} \left( \log \frac{x^+}{x_0^+} + 1 \right) + \frac{M}{\lambda}, \quad (6)$$

where  $\Delta = -\frac{N}{48\lambda^2 x_0^+}$ . It can be seen that the curvature singularity and the apparent horizon intersect at the point

$$x_{\text{int}}^+ = x_0^+ e^{\frac{4}{k}(\frac{M}{\lambda} - \alpha)}, \quad (7)$$

$$x_{\text{int}}^+ = \frac{k}{4\lambda^2 x_0^+} \left[ 1 - e^{-\frac{4}{k}(\frac{M}{\lambda} - \alpha)} \right]. \quad (8)$$

where

$$\alpha = N \frac{a}{24} \left( 1 - \log N \frac{a}{24} \right). \quad (9)$$

The evaporating solution can be continuously matched to the static solution

$$e^{-2\phi} + N \frac{a}{12} \phi = -\lambda^2 x^+ (x^- + \Delta) - \frac{k}{4} \log(-\lambda^2 x^+ (x^- + \Delta)) + C, \quad (10)$$

where

$$C = \frac{k}{4} \left( \log \frac{k}{4} - 1 \right) + \alpha, \quad (11)$$

with the emission of a thunderpop at the intersection point with negative energy  $E_{\text{thunderpop}} = -\lambda \frac{k}{4}$ , independent of the initial state. The remnant geometry (10) coincides with the one obtained in the process originated by gravitational collapse.

### 3. THERMAL BATH REMOVAL IN THE EXPONENTIAL MODEL

The semiclassical theory of the exponential model [7] is given by the action

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left( \phi R + 4\lambda^2 e^{\beta\phi} - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right) + S_P + \frac{N\beta}{96\pi} \int d^2x \sqrt{-g} \left( \phi R + \beta (\nabla \phi)^2 \right), \quad (12)$$

where  $S_P$  is the Polyakov action. The classical limit

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left( \phi R + 4\lambda^2 e^{\beta\phi} - \frac{1}{2} \sum_{i=1}^N (\nabla f_i)^2 \right), \quad (13)$$

arises as a theory with the conformal extra symmetry

$$\begin{aligned} \delta g_{\mu\nu} &= -\epsilon \beta g_{\mu\nu}, \\ \delta \phi &= \epsilon. \end{aligned} \quad (14)$$

The action (12) contains the adequate counterterms in order to preserve this symmetry when we add the Polyakov action, in analogy to the BPP-RST models. In absence of matter and taking the boundary conditions  $t_{\pm} = 0$  we have the solution (in Kruskal gauge,  $2\rho = \beta\phi$ )

$$ds^2 = -\frac{dx^+ dx^-}{\frac{\lambda^2 \beta \gamma}{C} + C x^+ x^-}, \quad (15)$$

where  $\gamma = 1/\left(1 + \frac{N\beta}{12}\right)$ . If  $C < 0$  (and  $\lambda^2 < 0$ ) it represents a black hole in thermal equilibrium with mass  $M = \frac{2}{\beta\gamma\pi} \sqrt{C}$  and temperature  $T_H = \frac{\beta\gamma}{4} M$  proportional to its mass. We now introduce the boundary conditions (4),(5) and (in order to simplify calculations) the incoming shock-wave

$$T_{++} = \frac{1}{2x_0^+ \beta \gamma} \delta(x^+ - x_0^+). \quad (16)$$

With these ingredients and in the limit  $N\beta \gg 1$ , we get the solution (for  $x^+ > x_0^+$ )

$$ds^2 = -\frac{\left(\frac{x_0^+}{x^+}\right)^{1/2}}{\frac{\lambda^2\beta\gamma}{C} + Cx_0^+x^-\left(\log\frac{x^+}{x_0^+}\right)}dx^+dx^- . \quad (17)$$

One can check that the curvature singularity and the apparent horizon do not intersect each other at any finite point. In contrast with the BPP-RST models the asymptotically flat coordinate  $\sigma^+$  at past null infinity is not the same as for the initial solution implying that the incoming flux has not been actually removed. In fact, it can be seen that (with an adequate choice of the point  $x_0^+$ ) the incoming flux starts to decrease continuously at  $x^+ = x_0^+$ , vanishes at a certain time  $x_1^+$  and for  $x^+ > x_1^+$  becomes a small negative flux which goes to zero at infinity. The incoming flux cannot affect qualitatively the result because at late times it becomes negative and even so the black hole does not evaporate.

We notice that recently a procedure of thermal bath removal [8] has been applied to the Schwarzschild black hole by applying an operator formalism [9] to quantize the Vaidya solution.

#### 4. EXTRA SYMMETRIES

The action of the exactly solvable models (1) can be rewritten as follows

$$S = S_0 \left( \bar{g}_{\mu\nu}, e^{-2\phi} + \frac{N}{12} a e^{-2\phi} \right) + S_P(\bar{g}_{\mu\nu}), \quad (18)$$

where  $\bar{g}_{\mu\nu} = g_{\mu\nu} e^{-2\phi}$  is an auxiliary metric which is invariant under the symmetry transformation

$$\delta\phi = \epsilon e^{2\phi}, \quad (19)$$

$$\delta g_{\mu\nu} = 2\epsilon g_{\mu\nu} e^{2\phi}. \quad (20)$$

This simple reconstruction of the one-loop quantum corrected CGHS models suggest to investigate the existence of additional extra symmetries for generic 2D dilaton models to construct solvable semiclassical models.

The general 2D dilaton gravity action can be written (through a conformal reparametrization of the metric [10,11]) as:

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} (\phi R + V(\phi)). \quad (21)$$

In conformal gauge this action can be identified with a 2D sigma-model action [12,6]

$$S = \frac{1}{2\pi} \int d^2x \left( -4\partial_+\phi\partial_-\rho + \frac{1}{2}V(\phi)e^{2\rho} \right), \quad (22)$$

where  $(\rho, \phi)$  play the role of target-space coordinates and the target-space metric is flat. The condition for the existence of a symmetry in these models [12] gives in this case

$$\frac{d^2 \log V(\phi)}{d\phi^2} = 0, \quad (23)$$

which has the general solution  $V = 4\lambda^2 e^{\beta\phi}$  with  $\lambda, \beta$  constants. For  $\beta = 0$  we recover the CGHS model while for  $\beta \neq 0$  we have the exponential model.

By direct computation one can check that the action (21) possesses the following extra symmetries for an arbitrary potential

$$\delta_1\phi = 0,$$

$$\delta_1 g_{\mu\nu} = \epsilon_1 \left( \frac{g_{\mu\nu}}{(\nabla\phi)^2} - 2 \frac{\nabla_\mu\phi\nabla_\nu\phi}{(\nabla\phi)^4} \right), \quad (24)$$

$$\delta_2\phi = \epsilon_2,$$

$$\delta_2 g_{\mu\nu} = \epsilon_2 V \left( \frac{g_{\mu\nu}}{(\nabla\phi)^2} - 2 \frac{\nabla_\mu\phi\nabla_\nu\phi}{(\nabla\phi)^4} \right), \quad (25)$$

$$\delta_3\phi = 0,$$

$$\delta_3 g_{\mu\nu} = -\frac{\epsilon_3}{2} \left[ g_{\mu\nu} + J \left( \frac{g_{\mu\nu}}{(\nabla\phi)^2} - 2 \frac{\nabla_\mu\phi\nabla_\nu\phi}{(\nabla\phi)^4} \right) \right]. \quad (26)$$

where  $\frac{dJ}{d\phi} = V(\phi)$ , which close down to a non-abelian Lie algebra in which  $\delta_2$  is a central generator, but  $\delta_1$  and  $\delta_3$  generate the affine subalgebra

$$[\delta_1, \delta_3] = \frac{1}{2}\delta_1. \quad (27)$$

These three symmetries imply the existence of three free field equations which are

$$\square \int^\phi \frac{d\tau}{2E + J(\tau)} = 0, \quad (28)$$

$$R + \square \log(\nabla\phi)^2 = 0, \quad (29)$$

$$\square E = 0. \quad (30)$$

where  $E = \frac{1}{2} \left( (\nabla\phi)^2 - J(\phi) \right)$  is not only a free field but it is also a conserved quantity which can be interpreted as the local energy of the solutions.

The symmetry  $\delta = \delta_2 - 4\lambda^2\delta_1$  when  $V = 4\lambda^2$  is just the conformal symmetry (19),(20) and therefore generalizes this symmetry for an arbitrary model. Moreover  $\delta_\beta = \delta_2 + 2\beta\delta_3$  when  $V = 4\lambda^2 e^{\beta\phi}$  is exactly the conformal symmetry of the exponential model (14). It is possible to construct an auxiliary metric

$$\begin{aligned} \bar{g}_{\mu\nu} &= \frac{2E_\lambda}{(\nabla\phi)^2} g_{\mu\nu} \\ &+ \left( \frac{1}{2E_\lambda} - \frac{2E_\lambda}{(\nabla\phi)^4} \right) \nabla_\mu\phi \nabla_\nu\phi, \end{aligned} \quad (31)$$

where  $E_\lambda = \frac{1}{2} \left( (\nabla\phi)^2 - J(\phi) \right) + 2\lambda^2\phi$ , which is invariant under the symmetry  $\delta$ . Relation (31) can be easily inverted to give

$$\begin{aligned} g_{\mu\nu} &= \frac{2\bar{E}_\lambda}{(\bar{\nabla}\phi)^2} g_{\mu\nu} \\ &+ \left( 2\bar{E}_\lambda - 2\frac{\bar{E}_\lambda}{(\bar{\nabla}\phi)^4} \right) \nabla_\mu\phi \nabla_\nu\phi. \end{aligned} \quad (32)$$

With the aid of this metric we can construct a semiclassical action invariant under  $\delta$  by coupling the matter conformally to this invariant metric

$$\begin{aligned} S &= S_{DG}(g(\bar{g}), \phi) \\ &- \frac{1}{2} \sum_{i=1}^N \int d^2x \sqrt{-\bar{g}} \bar{g}^{\mu\nu} \partial_\mu f_i \partial_\nu f_i \\ &+ S_P(\bar{g}), \end{aligned} \quad (33)$$

where  $S_{DG}$  is the action (21). This action is invariant under the transformation

$$\begin{aligned} \delta\phi &= \epsilon, \\ \delta g_{\mu\nu} &= 0, \end{aligned} \quad (34)$$

and is the natural generalization of the BPP model for an arbitrary model of 2D dilaton gravity.

## REFERENCES

1. C. G. Callan, S. B. Giddings, J. A. Harvey and A. Strominger, Phys. Rev. D45 (1992) 1005.
2. J. G. Russo, L. Susskind and L. Thorlacius, Phys. Rev. D46 (1993) 3444; Phys. Rev. D47 (1993) 533.
3. S. Bose, L. Parker and Y. Peleg, Phys. Rev. D52 (1995) 3512; Phys. Rev. Lett. 76 (1996) 861; *Validity of the Semiclassical Approximation and Back-Reaction*, WIS-MILW-96-th-10.
4. J. Cruz and J. Navarro-Salas, Phys. Lett. B 375 (1996) 47.
5. R. B. Mann, Nucl. Phys. B 418 (1994) 231
6. J. Cruz, J. Navarro-Salas, M. Navarro and C. F. Talavera, *Symmetries and Black Holes in 2D Dilaton Gravity*, hep-th/9606097.
7. J. Cruz and J. Navarro-Salas, Phys. Lett. B 387 (1996) 51
8. J. Cruz, A. Mikovic and J. Navarro-Salas, *A quantum model for Schwarzschild black hole evaporation*, hep-th/9611219
9. A. Mikovic and V. Radovanovic, *Two-Loop Back Reaction in 2D Dilaton Gravity*, hep-th/9606098 (to appear in Nucl. Phys. B).
10. T. Banks and M. O'Loughlin, Nucl. Phys. B362 (1991) 649.
11. J. Gegenberg, G. Kunstatter and D. Louis-Martinez, Phys. Lett. B321 (1994) 193; Phys. Rev. D51 (1995) 1781; *Classical and Quantum Mechanics of Black Holes in Generic 2D Dilaton Gravity*, gr-qc/9501017.
12. Y. Kazama, Y. Satoh and A. Tsuchiya, Phys. Rev. D51 (1995) 4265.